Procedural Variation
KS2

Mastery Specialist Teachers

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‘People are naturally curious, but we are not naturally good thinkers; unless the cognitive conditions are right, we will avoid thinking.’

Willingham, D.T. (2009) *Why Don’t Students Like School?*

Variation Theory in Practice

Find a friend. One of you will be 'A' and the other 'B'.

How did you get on? Compare the two sets of calculations.

What do you notice?

What's the same, what's different?

Set A	Set B
120 – 90	120 – 90
235 – 180	122 – 92
502 – 367	119 – 89
122 – 92	235 – 180
119 – 89	237 – 182
237 – 182	502 – 367

Taken from Mike Askew, Transforming Primary Mathematics, Chapter 6

Variation is not the same as Variety

- Variety
  - ‘Pick and mix’
  - Most practice exercises contain variety

- Variation
  - Careful choice of WHAT to vary
  - Careful choice of what the variation will draw attention to

Conceptual Variation

‘The central idea of teaching with variation is to highlight the essential features of the concept through varying the non- essential features.’

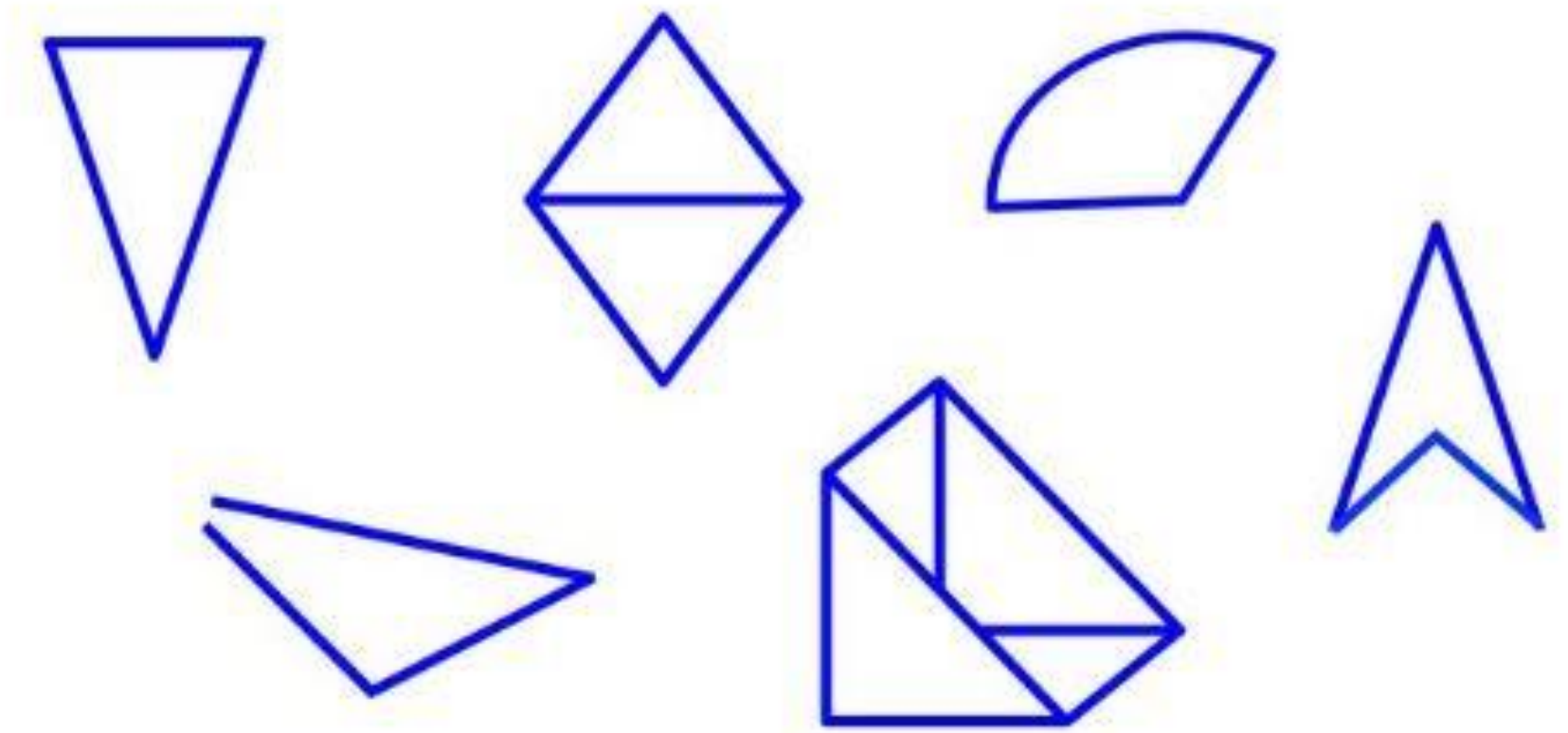
Gu, Huang & Marton, 2004

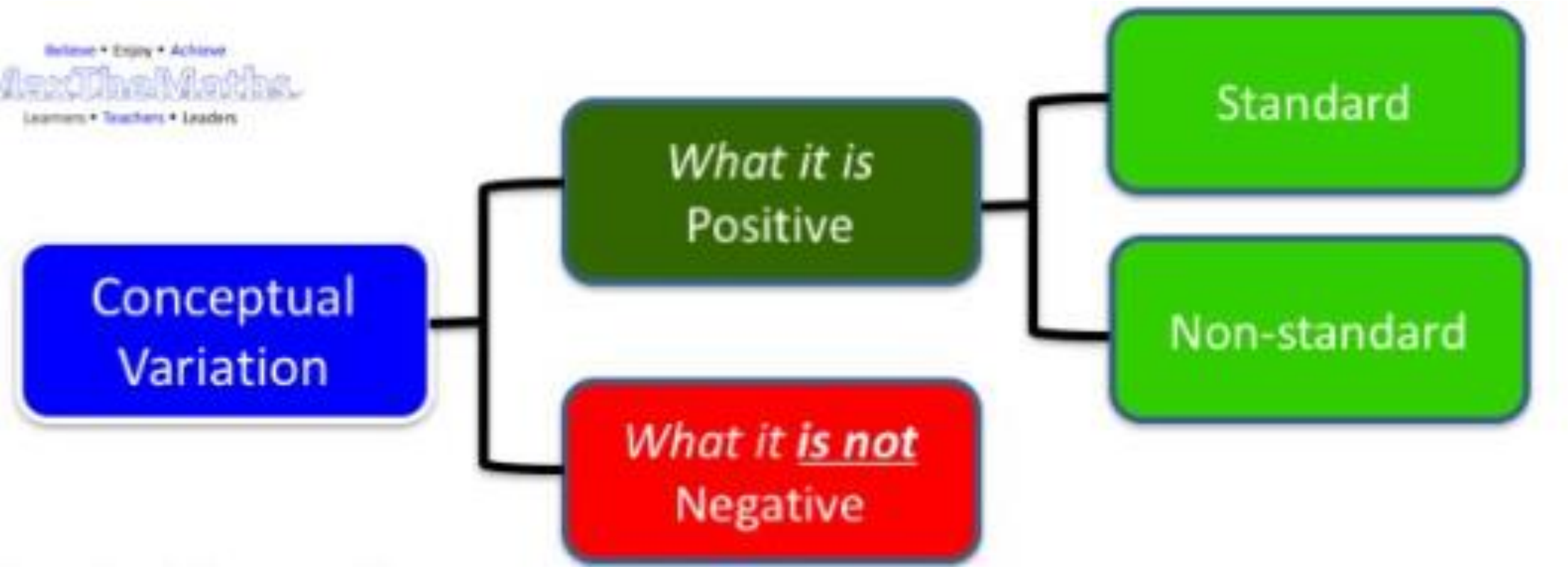
Conceptual and Non-Conceptual (Examples and non examples)

- An elephant
- A non-elephant

Conceptual and Non-Conceptual (Examples and non examples)

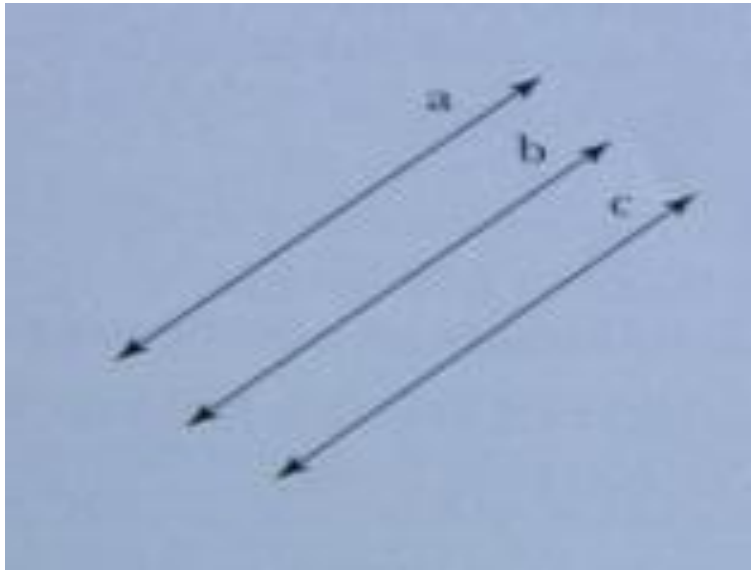
Triangle or Not a Triangle?



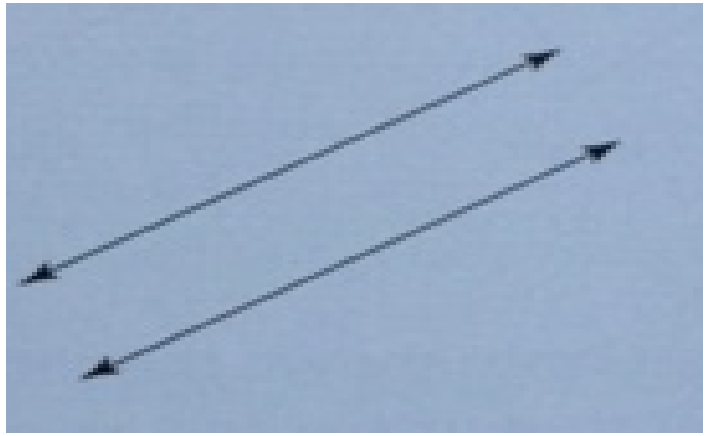


Standard and non-standard examples

11 year olds were asked: Is line a parallel to line c?



Most answered, ‘No, because line b is in the way.’

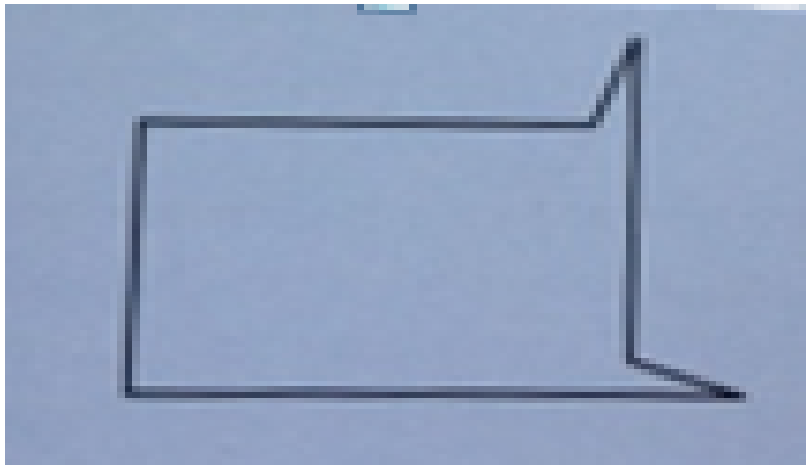


The concept of parallel lines is almost always illustrated like this.

Boaler, Jo. (2016) Mathematical Mindsets

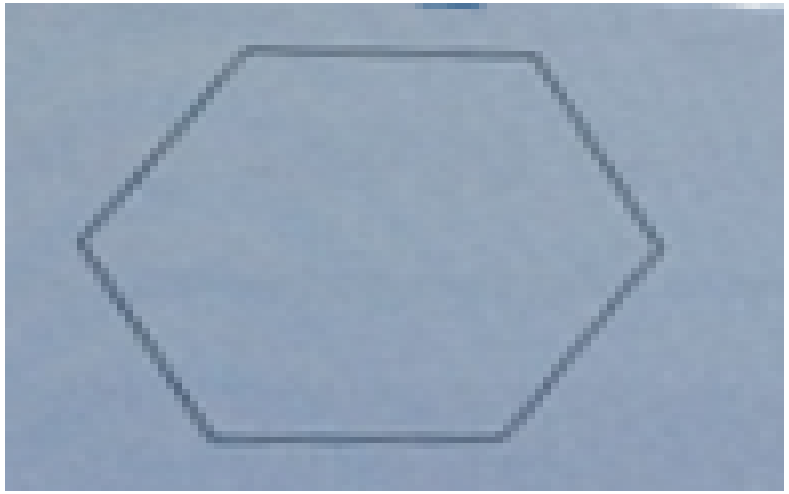
Standard and non-standard examples

They were asked to name this shape:



Most were unable to …

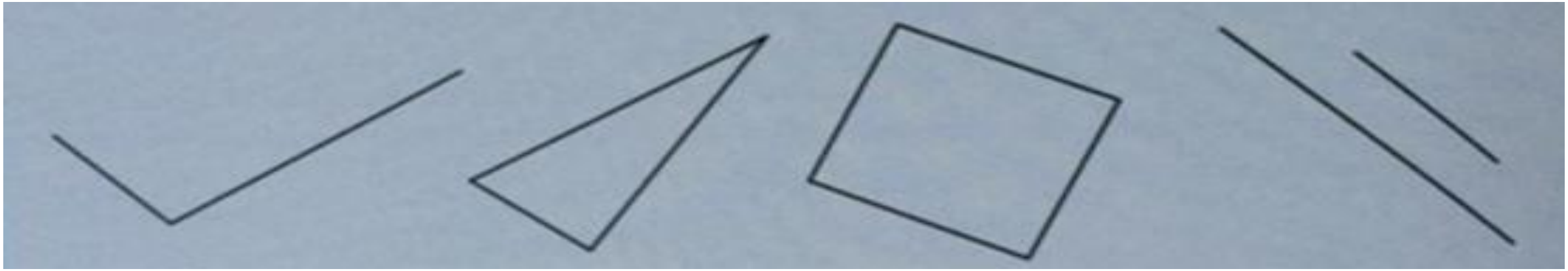
because hexagons are usually shown like this:



Boaler, Jo. (2016) Mathematical Mindsets

Standard and non-standard examples

Over half of eight year olds did not see these as examples of a right angle, triangle, square or parallel lines.



The images they were used to seeing had limited their understanding.

Boaler, Jo. (2016) Mathematical Mindsets

POG

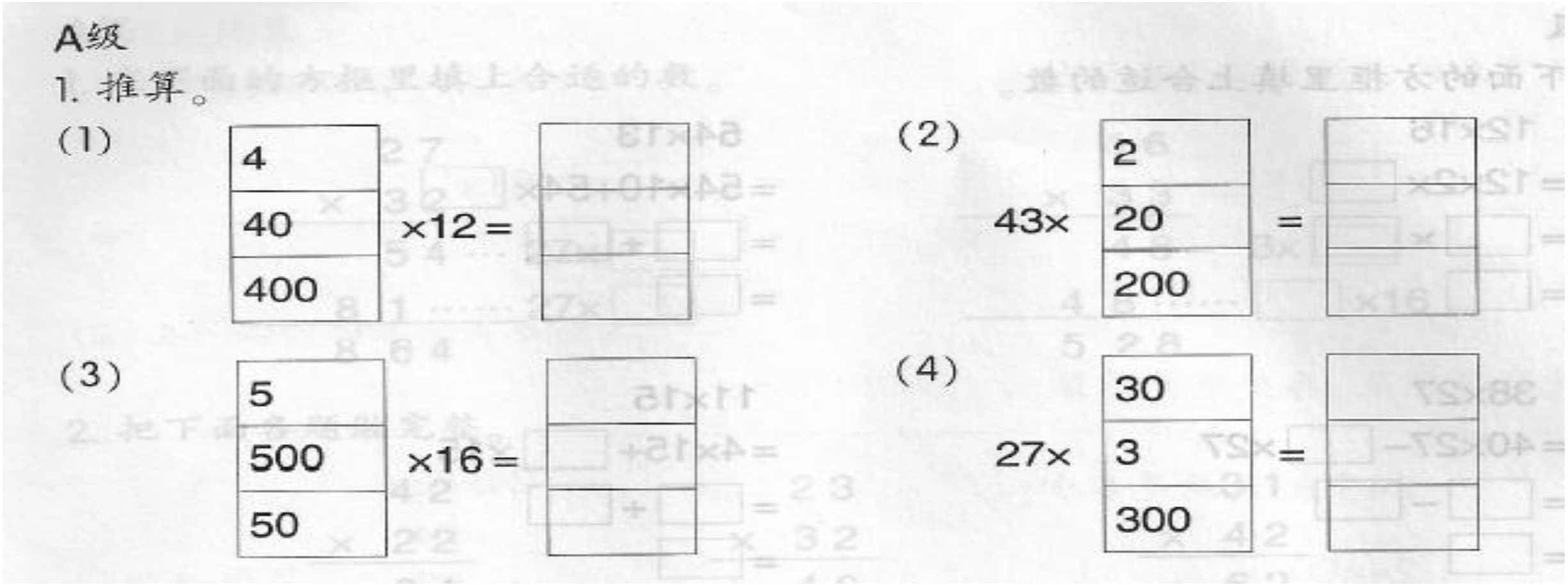
Peculiar

Ordinary

General

Procedural Variation

Variation


A级

1. 推算。

(1)

4
40
400

×12 = | |

(2) 43× | 2 |
| 20 |
| 200 | = | |

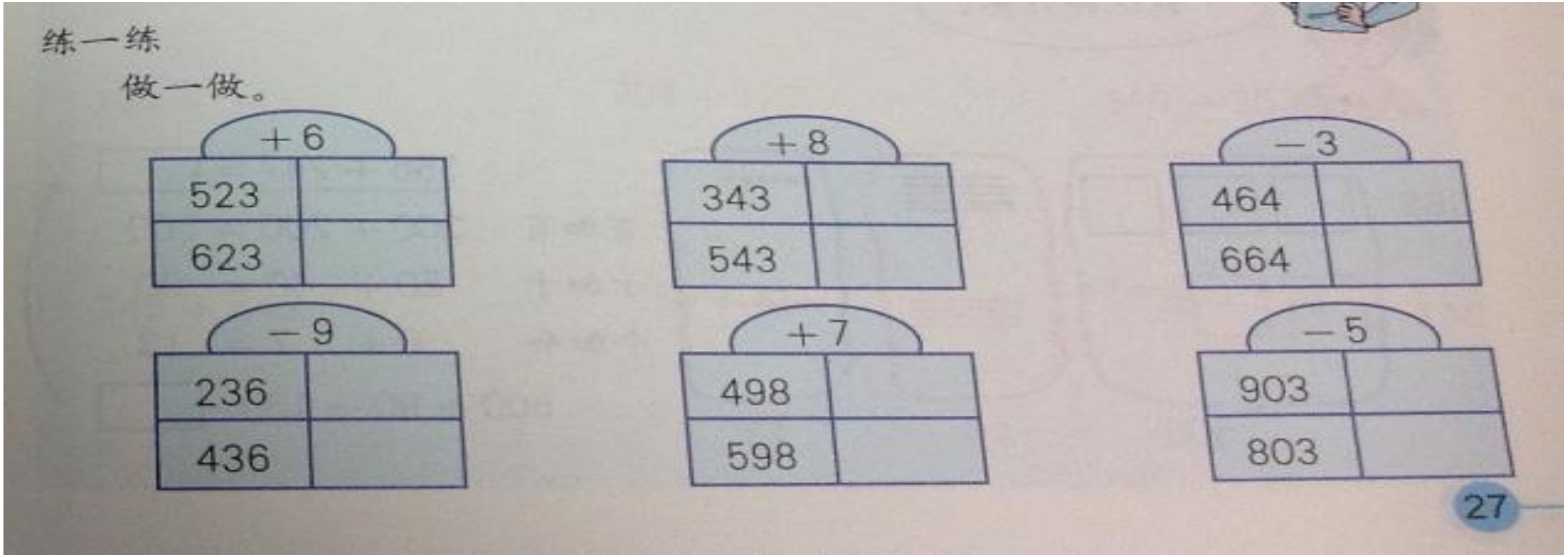
(3)

5
500
50

×16 = | |

(4) 27× | 30 |
| 3 |
| 300 | = | |

What is the variation within the questions and between the questions



Procedural Variation:

- Is dynamic; where I move between one calculation and the next there is a connection.
- Provides the opportunity
  - ❖ to focus on relationships, not just the procedure
  - ❖ to make connections between problems using one problem to work out the next
- Should not be repetitive in a way that leads children to stop thinking.

Children need to be taught from an early age to look for and recognise these connections.

What do you notice?

What's the same?

What's different?

‘In designing [these] exercises, the teacher is advised to avoid mechanical repetition and to create an appropriate path for practising the thinking process with increasing creativity.’

Gu, 1991

Mastery – Deep Understanding

Level of challenge:

1) $\frac{5}{7} = \frac{\quad}{14} = \frac{15}{\quad} = \frac{20}{\quad} = \frac{25}{\quad} = \frac{30}{\quad} = \frac{35}{\quad}$

2) $\frac{3}{8} = \frac{6}{\quad} = \frac{\quad}{24} = \frac{\quad}{32} = \frac{\quad}{40} = \frac{18}{\quad} = \frac{21}{\quad}$

3) $\frac{2}{5} = \frac{4}{\quad} = \frac{6}{\quad} = \frac{8}{\quad} = \frac{\quad}{25} = \frac{12}{\quad} = \frac{14}{\quad}$

$\frac{5}{7} = \frac{?}{?}$ $\quad \frac{16}{36} = \frac{?}{?}$ $\quad \frac{4}{8} = \frac{12}{?}$ $\quad \frac{3}{5} = \frac{?}{40}$

$\frac{3}{?} = \frac{21}{63}$ $\quad \frac{20}{32} = \frac{8}{?}$ $\quad \frac{?}{11} = \frac{?}{88}$ $\quad \frac{48}{64} = \frac{?}{48}$

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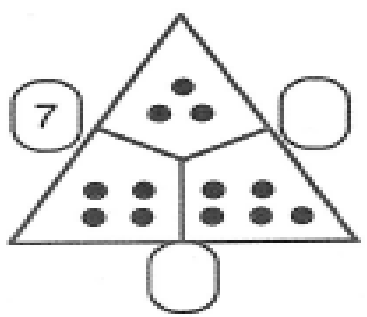
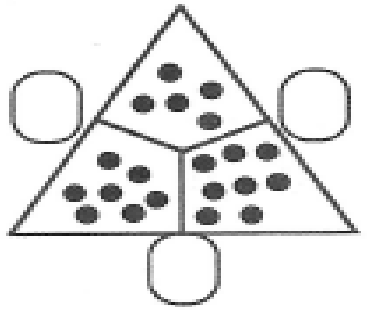
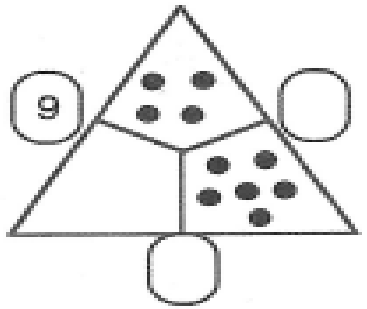
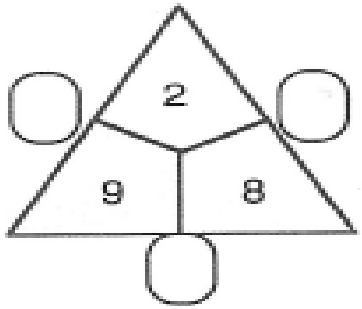
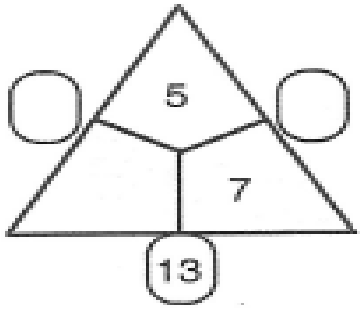
Procedural Variation
Focusing on relationships – going deeper

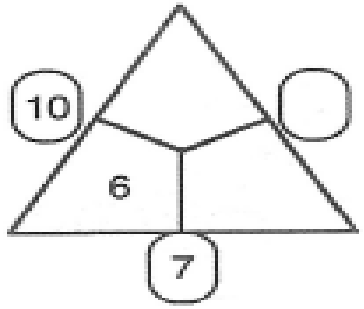




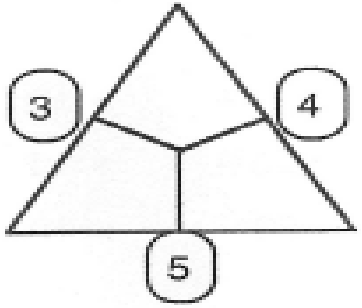


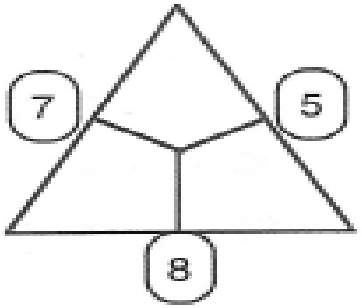






(3)* 用小圆片摆一摆，试一试。

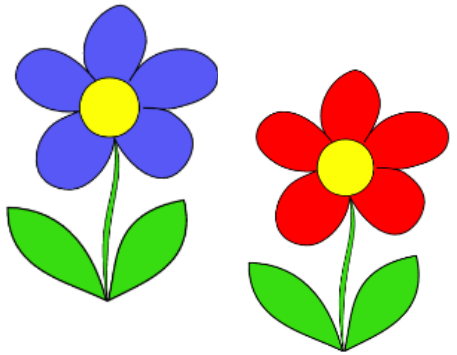




Procedural Variation
Problem solving – going deeper

- There are 3 red flowers and 5 blue flowers. How many flowers are there altogether?
- There are 8 flowers; some are red and some are blue.

  If three of them are red, how many are blue?
- There are 8 flowers; some are red and some are blue. If there are two more blue flowers than red flowers, how many are there of each colour?

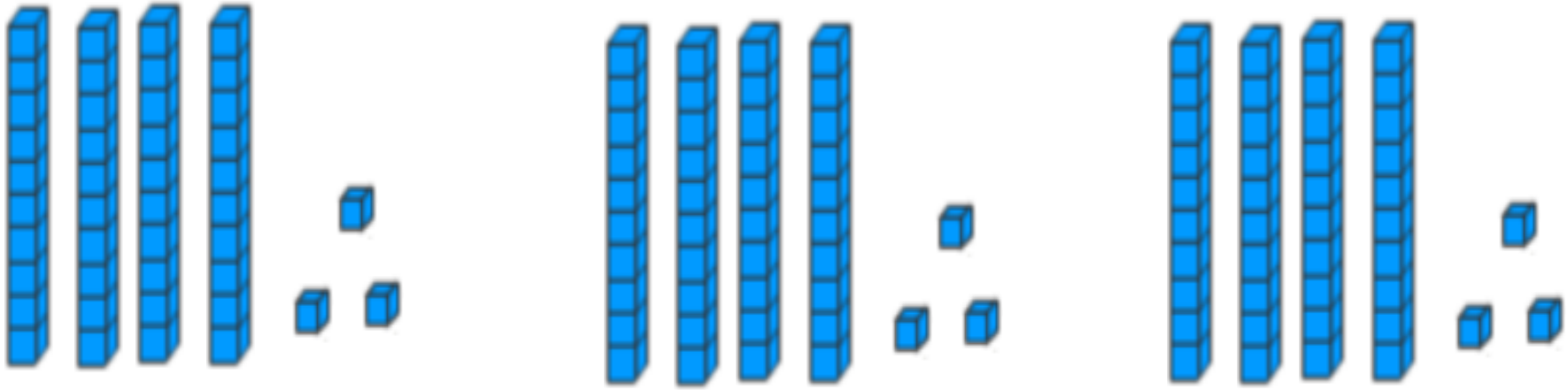


‘It is not enough to carry the sequence on downwards and upwards, going with the grain, as it were. They also need to articulate what they think is going on horizontally *across* the grain.’

Anne Watson and John Mason

Variation and Mathematical Structure

MT 194, 2006



43 X 3

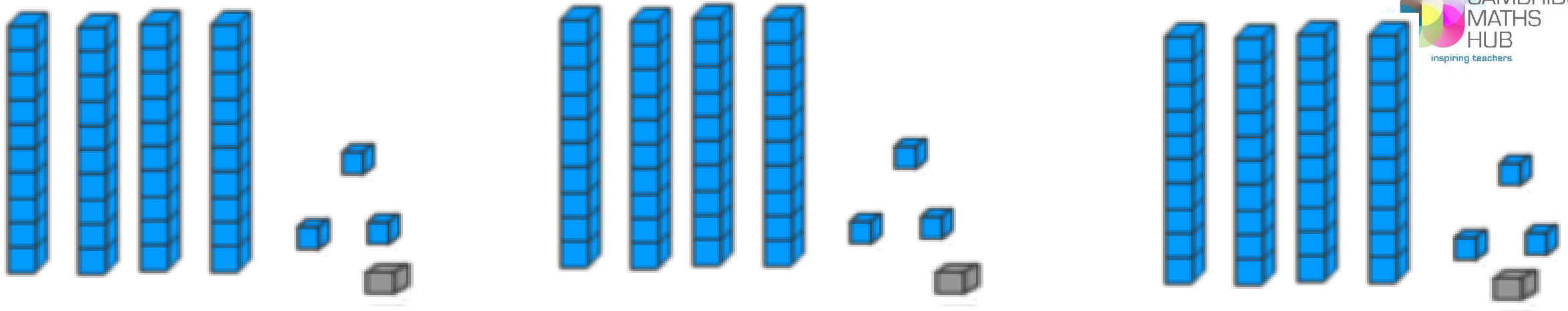
Which number is the multiplicand?
Which number is the multiplier?

STEM SENTENCES:

The multiplicand is …

The multiplier is …

43 X 3 = 129

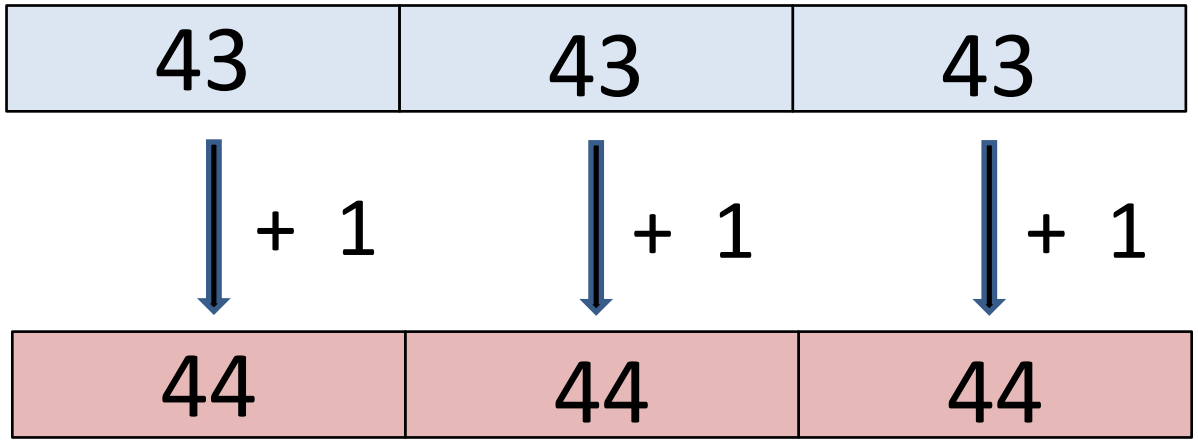


$43 \times 3 = 129$

44×3

What's the same and what's different about these two calculations?

Can you use your answer from 43 x 3 to work out 44 x 3?



43 X 3 = 129

44 X 3 = ?

44 X 3 = 43 x 3 + ?

A full jar of beads holds 58 beads. How many beads are there in 6 full jars?

58 X 6

STEM SENTENCES: The multiplicand is …
The multiplier is …

58	58	58	58	58	58

58 X 6 = 348

A full jar of beads holds 58 beads. In 6 full jars there are 348 beads.

58 X 6 = 348

58	58	58	58	58	58

We're going to use this answer to find out how many beads there are in 7 full jars.

Draw a bar picture to show 58 x 6

Draw a bar picture to show 58 x 7

What's the same and what's different about them?

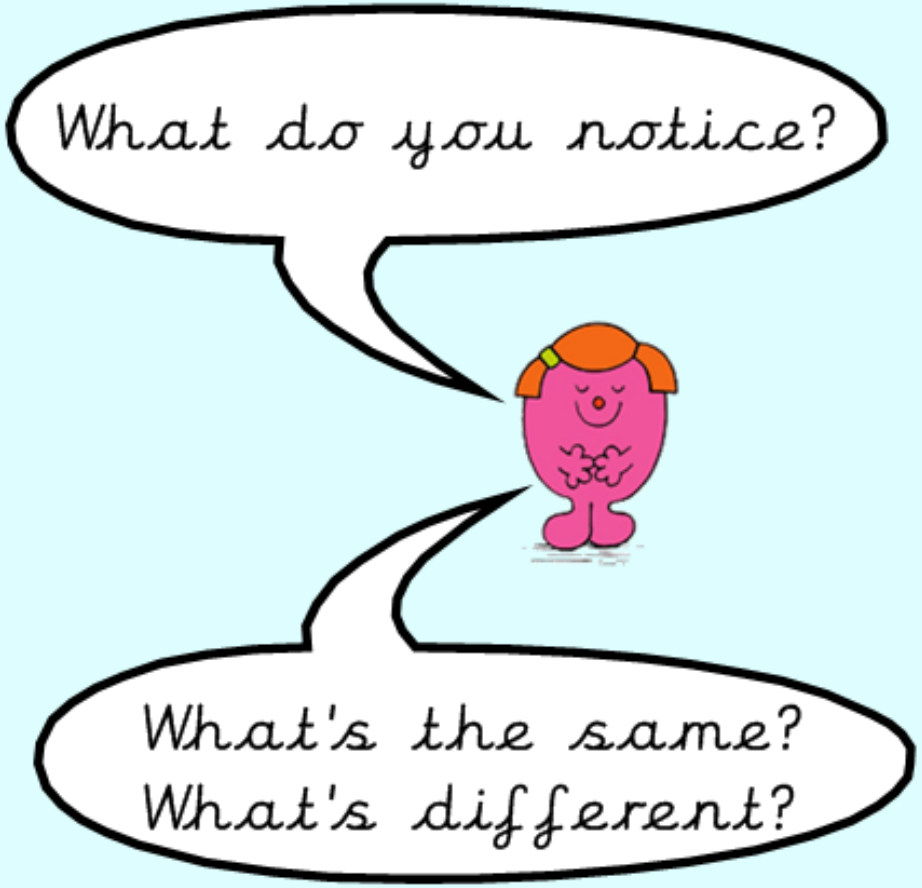
58	58	58	58	58	58

58	58	58	58	58	58	58

58 X 7 = 58 x 6 + ?

58 X 7 = 348 + 58 = 406

Try these:



$8 \times 7 = (8 \times 4) + (8 \times \square)$

$8 \times 8 = (8 \times 4) + (8 \times \square)$

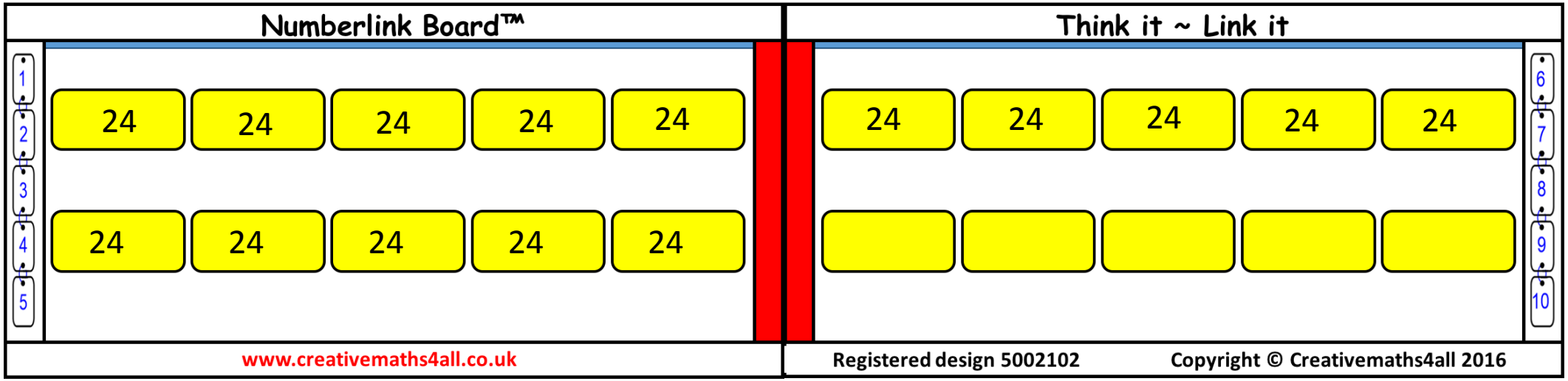
$8 \times \square = (8 \times 4) + (8 \times 6)$

$8 \times 10 > (8 \times 4) + (8 \times \square)$

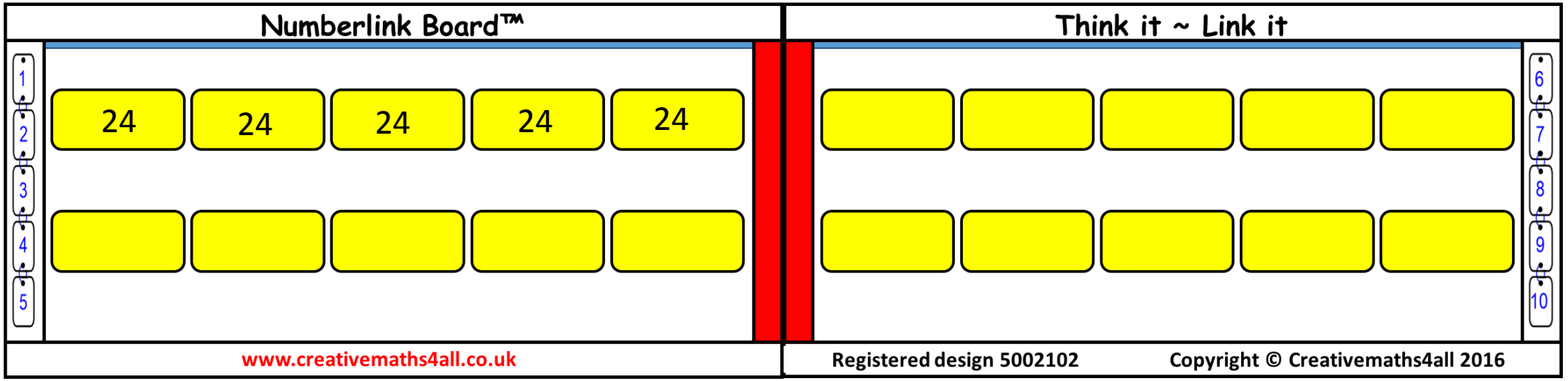
$80 \times 8 = (80 \times 5) + (\square \times 3)$

If the product of 24 and 15 is 360, what is the product of 24 and 16?

This is the same as 24 multiplied by sixteen or sixteen lots of 24.



If the product of 24 and 5 is 120, what is the product of 124 and 5? Can you explain why?



If the product of 24 and 15 is 360, what is the product of 124 and 15? Can you explain why?

Exposing and using structure

$$8 - 5 = 3$$

$$9 - 6 = 3$$

$$10 - 7 = 3$$

- These calculation all have a difference of 3. Can you explain why?

- Application of understanding this structure can be very powerful. "I can build on and apply my understanding."

$$14.3 - 3.8 = 14.5 - 4 = 10.5$$

‘So your memory is not a product of what you want to remember, or what you try to remember, it’s actually a product of what you think about.

Memory, therefore, can be seen as the residue of thought.’

Further Reading

Teaching with Procedural Variation: A Chinese Way of Promoting Deep Understanding of Mathematics
The examples in this article are in the main examples from Secondary Mathematics; however the introduction is helpful in exploring the idea of "variation" a key strategy that supports deep conceptual learning and mastery of mathematics in East Asian countries
http://www.cimt.plymouth.ac.uk/Journal/lai.pdf
Lai, M. Y., & Murray, S. (2012). Teaching with procedural variation: A Chinese way of promoting deep understanding of mathematics. *International Journal of Mathematics Teaching and Learning*.

The structures, goals and pedagogies of " variation problems" in the topic of addition and subtraction of 0-9 in Chinese textbooks and reference books.
This article also explores variation and draws on examples from Primary textbooks
http://repository.umac.mo/dspace/bitstream/10692/550/1/9441_0_WG13_Sun.pdf
Sun, X. H. (2013, April). The structures, goals and pedagogies of" variation problems" in the topic of addition and subtraction of 0-9 in Chinese textbooks and reference books. In *Eighth Congress of European Research in Mathematics Education (CERME 8), Apr. 2013.*

Gu, L, Huang, R, & Marton, F. (2004). **Teaching with variation: A Chinese way of promoting effective mathematics learning.** In Fan, L, Wong, NY, Cai, J, & Li, S (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (p. 309-347). Singapore: World Scientific

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